

Free fall temperature of Schwarzschild-Tangherlini-AdS black hole

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Investigating five-dimensional Schwarzschild-Tangherlini-AdS black hole, we construct a (6+2)-dimensional flat embedding structure. Exploiting this flat manifold, we evaluate a free fall temperature of the black hole in manifold ranging from outer event horizon to the infinity.

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I. INTRODUCTION

Since Schwarzschild-Tangherlini black hole was proposed [1], considerable progress has been made in higher dimensional general relativity. The higher dimensional gravity was exploited to describe the black hole entropy for a five-dimensional black hole in string theory by counting the degeneracy of BPS soliton bound states [2]. The five-dimensional black hole theory admits a rotating black ring which is a stationary asymptotically flat vacuum solution associated with an event horizon of non-spherical topology [3]. Moreover, the ten-dimensional black hole possesses dynamical instabilities of even horizons [4]. On the other hand, (2+1) dimensional Bañados-Teitelboim-Zanelli (BTZ) black hole [5–9] was proposed and it is related with the string theory in ten-dimensional spacetime. Moreover, a slightly modified solution of the BTZ black hole produces a solution to the string theory, so-called the black string [10–12]. This black string solution is known to be a solution to the lowest order β -function equation associated with quantum corrections [13].

One of the features of solutions to the field equations of general relativity is the presence of singularities. As a novel way to remove the coordinate singularities, higher dimensional flat embedding of the black hole solution is a subject of interest to both mathematicians and physicists. In differential geometry, the four-dimensional Schwarzschild metric [14] is well known not to be embedded in R^5 [15]. For the Schwarzschild black hole, a (5+1)-dimensional flat embedding was constructed [16–18] to remove the coordinate singularity and to study a thermal Hawking effect on the curved manifold [19, 20] related with the Unruh effect [21] in the higher dimensional manifold. Recently, a free fall temperature of the Schwarzschild-AdS black hole was evaluated in the higher dimensional flat spacetime [22]. Later, the free fall temperature for the Gibbons-Maeda-Garfinkle-Horowitz-Strominger spacetime [23, 24] was evaluated [25].

In this paper, we will construct the free fall temperature for the Schwarzschild-Tangherlini-AdS black hole. To do this, we will construct (6+2)-dimensional flat embedding. This paper is organized as follows: In Section II, we will explicitly evaluate the free fall temperature of the Schwarzschild-Tangherlini black hole, and in Section III we will derive the free fall temperature for the Schwarzschild-Tangherlini-AdS black hole. Section IV includes a summary and discussions.

II. FREE FALL TEMPERATURE OF SCHWARZSCHILD-TANGHERLINI BLACK HOLE

In order to investigate the free fall temperature of the five-dimensional Schwarzschild-Tangherlini black hole defined on the total manifold $S^3 \times \mathbf{R}^2$, we consider a five-metric of the form

$$ds^2 = -N^2 dt^2 + N^{-2} dr^2 + r^2 d\Omega_3^2, \quad (2.1)$$

where the lapse function is given by [26]

$$N^2 = 1 - \frac{\mu}{r^2}, \quad (2.2)$$

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and

$$d\Omega_3^2 = d\alpha^2 + \sin^2 \alpha (d\theta^2 + \sin^2 \theta d\phi^2). \quad (2.3)$$

Here Ω_3 is the solid angle in the three-dimensional compact sphere S^3 whose value is $2\pi^2$. Here, we have three angles of the three-sphere whose ranges are defined by $0 \leq \alpha \leq \pi$, $0 \leq \theta \leq \pi$ and $0 \leq \phi \leq 2\pi$. In the lapse function N , we have the five-dimensional Schwarzschild-Tangherlini radius μ defined as [26]

$$\mu = \frac{8GM}{3\pi}, \quad (2.4)$$

with the black hole mass M . Here we observe that in the weak gravity limit the gravitational potential in the five-dimensional theory is proportional to $1/r^2$, different from the $1/r$ of the four-dimensional Newtonian theory.

Now, we evaluate the event horizon radius r_H by exploiting the vanishing lapse function at $r = r_H$ to yield

$$r_H = \mu^{1/2}, \quad (2.5)$$

and the surface gravity κ is given by

$$\kappa = \frac{1}{2} \frac{dN^2}{dr} \Big|_{r=r_H} = \frac{1}{r_H}. \quad (2.6)$$

The Hawking temperature is then given by

$$T_H = \frac{1}{2\pi r_H}. \quad (2.7)$$

After some algebra, using embedding isometry for the five-dimensional Schwarzschild-Tangherlini black hole we construct the (6+1)-dimensional flat embedding structure [27]

$$ds^2 = \eta_{AB} dz^A dz^B, \quad (2.8)$$

where the (6+1) flat metric is given by

$$\eta_{AB} = \text{diag} (-1, +1, +1, +1, +1, +1, +1) \quad (2.9)$$

with the coordinate transformations

$$\begin{aligned} z^0 &= \kappa^{-1} \left(1 - \frac{\mu}{r^2}\right)^{1/2} \sinh \kappa t, \\ z^1 &= \kappa^{-1} \left(1 - \frac{\mu}{r^2}\right)^{1/2} \cosh \kappa t, \\ z^2 &= \int dr \left[\frac{r_H^2 (r^3 + r_H r^2 + r_H^2 r + r_H^3)}{r^4 (r + r_H)} \right]^{1/2}, \\ z^3 &= r \sin \alpha \sin \theta \cos \phi, \\ z^4 &= r \sin \alpha \sin \theta \sin \phi, \\ z^5 &= r \sin \alpha \cos \theta, \\ z^6 &= r \cos \alpha. \end{aligned} \quad (2.10)$$

Now we assume that an observer at rest is freely falling from the radial position $r = r_0$ at $\tau = 0$ [22, 25]. The equations of motion for the orbit of the observer are given as follows

$$\begin{aligned} \frac{dt}{d\tau} &= \frac{(1 - x_0)^{1/2}}{1 - x}, \\ \frac{dr}{d\tau} &= -(x - x_0)^{1/2}, \end{aligned} \quad (2.11)$$

where the dimensionless variable x is defined as

$$x = \frac{\mu}{r^2}. \quad (2.12)$$

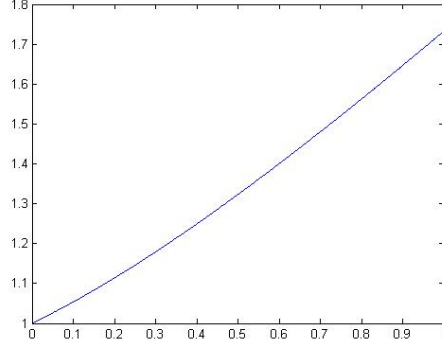


FIG. 1: T_{FFAR}/T_H in terms of the dimensionless variable $x = \mu/r^2$.

Exploiting Eqs. (2.10) and (2.11), we obtain a_7 acceleration:

$$a_7^2 = \frac{1}{\mu}(1 + x + x^2). \quad (2.13)$$

Using the definition of the freely falling observer at rest (FFAR) temperature

$$T_{FFAR} = \frac{a_7}{2\pi}, \quad (2.14)$$

we end up with

$$T_{FFAR}^2 = \frac{1}{4\pi^2\mu}(1 + x + x^2). \quad (2.15)$$

In Figure 1, we depict the ratio of the free fall temperature T_{FFAR} and the Hawking one T_H as a function of the dimensionless variable x defined in Eq. (2.12). Here one notes that, at $x = 0$ corresponding to the asymptotic region, the free fall temperature approaches to the Hawking temperature, as expected. At the event horizon $r = \mu^{1/2}$, we find the temperature $T_{FFAR} = \sqrt{3}T_H$. This value is different from that in the four-dimensional Schwarzschild black hole case where the free fall temperature is given by $T_{FFAR}^{4d} = 2T_H^{4d}$ [22]. Next, we obtain the local fiducial temperature corresponding to the Unruh temperature for the Schwarzschild-Tangherlini black hole:

$$T_{FID}^2 = \frac{1}{4\pi^2\mu(1-x)}. \quad (2.16)$$

III. FREE FALL TEMPERATURE OF SCHWARZSCHILD-TANGHERLINI-ADS BLACK HOLE

In this section, we will derive the free fall temperature for the Schwarzschild-Tangherlini-AdS black hole. To do this, we first construct the higher dimensional flat embedding of this black hole. We next exploit the orbit equations for the black hole to obtain the corresponding free fall temperature. We start with the Schwarzschild-Tangherlini-AdS black hole described by the lapse function [28]

$$N^2 = 1 - \frac{\mu}{r^2} + \frac{r^2}{l^2}, \quad (3.1)$$

where $\Lambda = l^{-2}$ is the cosmological constant. The lapse function can be also written in terms of the outer horizon r_+ as follows

$$N^2 = \frac{(r^2 - r_+^2)(r^2 + r_+^2 + l^2)}{l^2 r^2}. \quad (3.2)$$

Adopting the dimensionless variables

$$x = \frac{r_+^2}{r^2}, \quad c = \frac{l}{r_+}, \quad (3.3)$$

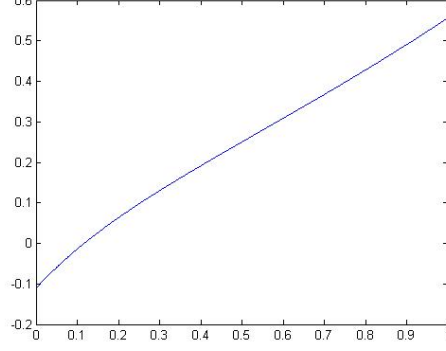


FIG. 2: T_{FFAR}^2/T_H^2 for $c = 1$ in terms of the dimensionless variable $x = \mu/r^2$.

we rewrite the lapse function (3.2) as follows

$$N^2 = \frac{(1-x)[1+(c^2+1)x]}{c^2x}. \quad (3.4)$$

After some algebraic manipulation, we newly find the (6+2)-dimensional flat embedding with the metric of the form

$$\eta_{AB} = \text{diag} (-1, +1, +1, +1, +1, +1, +1, -1) \quad (3.5)$$

associated with the coordinate transformations

$$\begin{aligned} z^0 &= \kappa^{-1} \left(1 - \frac{\mu}{r^2} + \frac{r^2}{l^2} \right)^{1/2} \sinh \kappa t, \\ z^1 &= \kappa^{-1} \left(1 - \frac{\mu}{r^2} + \frac{r^2}{l^2} \right)^{1/2} \cosh \kappa t, \\ z^2 &= \int dr \left[\frac{l^2 r_+^2 (r^3 + r_+ r^2 + r_+^2 r + r_+^3)}{r^4 (r + r_+) (r^2 + r_+^2 + l^2)} \right]^{1/2}, \\ z^3 &= r \sin \alpha \sin \theta \cos \phi, \\ z^4 &= r \sin \alpha \sin \theta \sin \phi, \\ z^5 &= r \sin \alpha \cos \theta, \\ z^6 &= r \cos \alpha, \\ z^7 &= \int dr \left[\frac{\kappa^{-2} (r^4 + 3r_+^4 + 2l^2 r_+^2) (r^3 + r_+ r^2 + r_+^2 r + r_+^3) + l^2 r^4 (r + r_+) (r^2 + r_+^2)}{l^2 r^4 (r + r_+) (r^2 + r_+^2 + l^2)} \right]^{1/2}, \end{aligned} \quad (3.6)$$

where the surface gravity for the Schwarzschild-Tangherlini-AdS black hole is given by

$$\kappa = \frac{2r_+^2 + l^2}{l^2 r_+} \quad (3.7)$$

Moreover, the Hawking temperature is given by

$$T_H = \frac{2r_+^2 + l^2}{2\pi l^2 r_+}. \quad (3.8)$$

Following the algorithm for finding the free fall temperature discussed above, we construct the equations of the orbit for the free fall observer at rest at $r = r_0$

$$\begin{aligned} \frac{dt}{d\tau} &= \frac{cx(1-x_0)^{1/2}[1+(c^2+1)x_0]^{1/2}}{x_0^{1/2}(1-x)[1+(c^2+1)x]}, \\ \frac{dr}{d\tau} &= - \left[\frac{(1-x_0)[1+(c^2+1)x_0]}{c^2 x_0} - \frac{(1-x)[1+(c^2+1)x]}{c^2 x} \right]^{1/2}. \end{aligned} \quad (3.9)$$

Exploiting the coordinate transformations (3.6) and the orbit equations (3.9), after some tedious algebra we arrive at the free fall temperature for the Schwarzschild-Tangherlini-AdS black hole:

$$T_{FFAR}^2 = \frac{-1 + (c^2 + 1)(c^2 + 3)x + (c^2 + 1)^2(1 + x)x^2}{4\pi^2 l^2 [1 + (c^2 + 1)x]}. \quad (3.10)$$

Here we note that in the vanishing c limit the free fall temperature is reducible to that of the Schwarzschild-Tangherlini black hole in Eq. (2.15). Moreover, in the very large c limit, the free fall temperature becomes

$$\frac{T_{FFAR}^2}{T_H^2} = 1 + x + x^2, \quad (3.11)$$

where T_H is the Hawking temperature given in Eq. (3.8). Figure 2 shows the ratio of T_{FFAR}^2 and T_H^2 for the case of $c = 1$ as a function of the dimensionless variable x . Finally, we evaluate the local fiducial temperature corresponding to the Unruh temperature for the Schwarzschild-Tangherlini-AdS black hole as follows

$$T_{FID}^2 = \frac{(2 + c^2)^2 x}{4\pi^2 l^2 (1 - x)[1 + (1 + c^2)x]}. \quad (3.12)$$

IV. CONCLUSIONS

In summary, we have constructed the (6+2)-dimensional flat embedding space for the five-dimensional Schwarzschild-Tangherlini-AdS black hole. Using this flat embedding manifold, we have calculated the free fall temperature of the black hole in manifold ranging from outer event horizon to the infinity.

Now we have couple of comments to address. In the coordinate transformations (2.10) and (3.6), there exist no coordinate singularities at event horizons $r = r_H$ and $r = r_+$, respectively. The free fall temperatures in Eqs. (2.15) and (3.10) are well defined without any divergence features. However, the local fiducial temperature in Eq. (2.16) for the Schwarzschild-Tangherlini black hole diverges at the event horizons $r = r_H$ corresponding to $x = 1$. As r approaches infinity, the local fiducial temperature becomes the Hawking temperature. For the Schwarzschild-Tangherlini-AdS black hole case, the local fiducial temperature in Eq. (3.12) vanishes in asymptotically flat spacetime and also diverges at the event horizons $r = r_+$.

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